



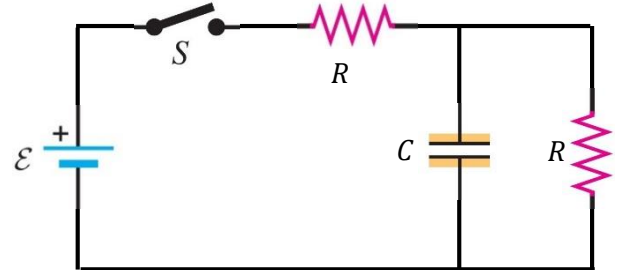
PHYS 102 – General Physics II Midterm Exam

Duration: 120 minutes

Saturday, 23 November 2024; 13:00

1. In the given circuit the switch S has been closed for a very long time.

- (a) (5 Pts.) What is the current through the battery?
- (b) (5 Pts.) What is the charge on the capacitor?
- (c) (5 Pts.) What is the energy stored in the capacitor?
- (d) (5 Pts.) At time $t = 0$, the switch S opened. What is the charge on the capacitor as a function of time?
- (e) (5 Pts.) How long does it take the capacitor to lose half of the energy initially stored in it?



Note: Write your answers in the boxes provided. Only the contents of the boxes will be graded.

If the switch is closed for a very long time, the capacitor is fully charged and no current passes through it. Therefore, we have a simple single loop circuit with total resistance $2R$, and $\mathcal{E} = 2R I$

(a)

$$I = \frac{\mathcal{E}}{2R}$$

The voltage on the capacitor is equal to the voltage on the resistance R in series with it. This means $Q = CV_C = CIR = C(\mathcal{E}/2R)R$.

(b)

$$Q = \frac{1}{2}C\mathcal{E}$$

$$U = \frac{Q^2}{2C}$$

(c)

$$U = \frac{1}{8}C\mathcal{E}^2$$

When the switch is opened, the capacitor will discharge through the resistor in parallel with it, and $q(t) = Q_0 e^{-t/RC}$

(d)

$$q(t) = \frac{1}{2}C\mathcal{E}e^{-t/RC}$$

Since $U(0)/2 = U(t)$ and $U(t) = U(0)e^{-2t/RC}$, we have $e^{-2t/RC} = 1/2$.

(e)

$$t = \frac{RC}{2} \ln 2$$

2. A particle with charge q and mass m , initially moving along the x axis with speed v_0 , enters a region where a uniform magnetic field $\vec{B} = B_0 \hat{k}$, where $B_0 > 0$, extends over a width $x = \ell$, as shown in the figure. The particle is deflected a distance d in the $+y$ direction as it traverses the field.

(a) (5 Pts.) Determine whether the charge is positive or negative.

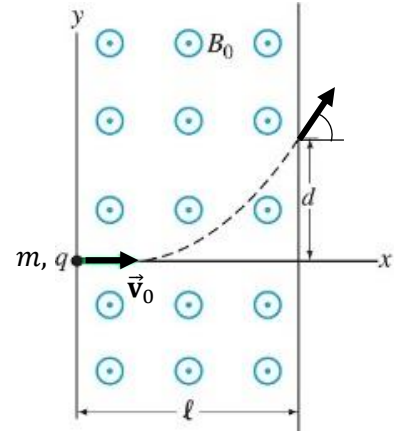
(b) (10 Pts.) What is the deflection d in terms of the quantities q, m, ℓ, v_0 and B_0 ?

(c) (10 Pts.) What is the velocity of the particle at the point (ℓ, d) where it exits the region?

Solution: (a) The force on the charge at the instant it enters the region is

$$\vec{F} = q (v_0 \hat{i}) \times (B_0 \hat{k}) \rightarrow \vec{F} = q v_0 B_0 (-\hat{j})$$

Since the charge is displaced in the $+y$ direction, it is negative.



(b) The particle will be moving along a circular path with constant speed while in the region. The radius of the circle can be found as

$$F = m a_R \rightarrow |q| v_0 B_0 = m \frac{v_0^2}{R} \rightarrow R = \frac{m v_0}{|q| B_0}.$$

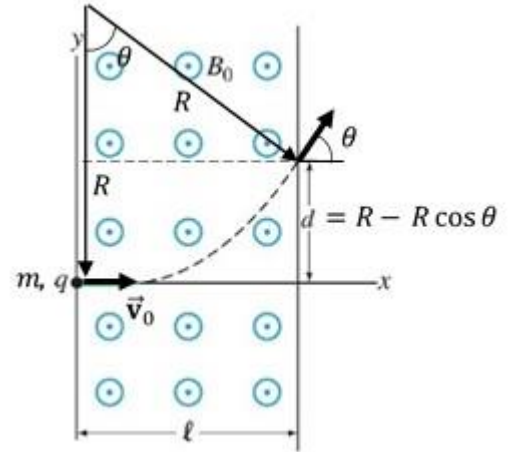
From the geometry of the figure, we have $d = R - R \cos \theta$.

Since

$$\sin \theta = \frac{\ell}{R} \rightarrow \cos \theta = \sqrt{1 - \frac{\ell^2}{R^2}},$$

we have

$$d = R - \sqrt{R^2 - \ell^2} \rightarrow d = \left(\frac{m v_0}{|q| B_0} \right) - \sqrt{\frac{m^2 v_0^2}{q^2 B_0^2} - \ell^2}.$$



(c) Since the speed of the particle is constant in the region, the velocity of the particle at the point (ℓ, d) where it exits the region is

$$\vec{v} = v_0 \cos \theta \hat{i} + v_0 \sin \theta \hat{j} \rightarrow \vec{v} = v_0 \sqrt{1 - \frac{\ell^2}{R^2}} \hat{i} + v_0 \left(\frac{\ell}{R} \right) \hat{j} \rightarrow \vec{v} = v_0 \sqrt{1 - \frac{q^2 B_0^2 \ell^2}{m^2 v_0^2}} \hat{i} + \left(\frac{|q| B_0 \ell}{m} \right) \hat{j}.$$

3. (25 Pts.) Find the magnetic field at the center of the square loop in the figure.

Solution:

Consider one side of the square and, using Biot-Savart law, find the magnetic field on the perpendicular bisector at a distance $d/2$ from the current.

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{r}}{r^2}, \quad d\vec{\ell} = dy \hat{j}, \quad r^2 = y^2 + \frac{d^2}{4}$$

$$\vec{r} = \frac{d}{2} \hat{i}, \quad \vec{r}' = y \hat{j}, \quad r = |\vec{r} - \vec{r}'| = \sqrt{y^2 + \frac{d^2}{4}}, \quad \hat{r} = \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|}$$

$$d\vec{\ell} \times \hat{r} = \frac{y (\hat{j} \times \hat{i})}{\sqrt{y^2 + d^2/4}} = \frac{y (-\hat{k})}{\sqrt{y^2 + d^2/4}}$$

$$d\vec{B} = \frac{\mu_0 I (d/2) (-\hat{k})}{4\pi} \int_{-d/2}^{d/2} \frac{dy}{(y^2 + d^2/4)^{3/2}}$$

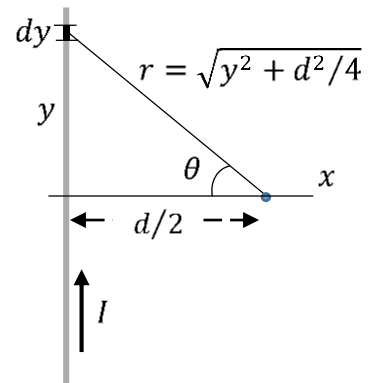
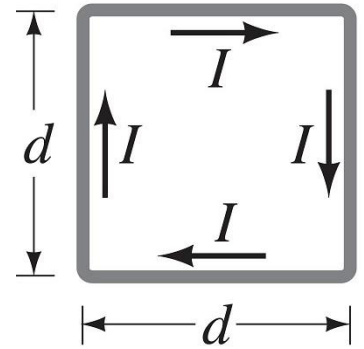
The integral can be evaluated using the substitution $y = \frac{d}{2} \tan \theta$. The result becomes

$$\vec{B} = \frac{\mu_0 I}{\sqrt{2}\pi d} (-\hat{k}),$$

The direction of the magnetic field is perpendicular to and into the plane of the page.

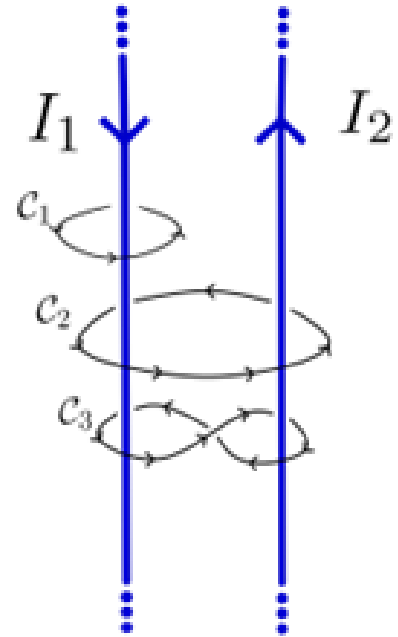
Since the given current is symmetric, the magnetic field at the center of the square loop can be found by multiplying the above result by four.

$$\vec{B} = \frac{2\sqrt{2}\mu_0 I}{\pi d} (-\hat{k})$$



4. Two infinitely long thin wires are parallel and carrying current in reverse directions as shown in the figure. The distance between the wires is d .

- (a) (8 Pts.) Using Ampere's law, find the magnitude of the magnetic field created by I_1 on the second wire.
 (b) (8 Pts.) Find the force per unit length between the wires. Is it attractive or repulsive?
 (c) (9 Pts.) Evaluate the line integral $\oint_C \vec{B} \cdot d\vec{l}$ over the closed paths C_2 and C_3 shown in the figure.



Solution: (a) Apply Ampère's law, integrating around the curve C_1 , which is a circle of radius r , centered at the current I_1

$$\oint_{C_1} \vec{B} \cdot d\vec{l} = \mu_0 I_1 \rightarrow (2\pi r)B(r) = \mu_0 I_1 \rightarrow B(d) = \frac{\mu_0 I_1}{2\pi d}$$

(b) The magnetic field created by I_1 at the position of I_2 is perpendicular to the direction of the current, out of the plane of the page. Therefore,

$$\vec{F} = I_2 \vec{\ell} \times \vec{B} = I_2 \ell B(d) \rightarrow F = I_2 \ell \left(\frac{\mu_0 I_1}{2\pi d} \right) \rightarrow \frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

and is repulsive.

(c) According to the right hand rule:

$$\oint_{C_2} \vec{B} \cdot d\vec{l} = \mu_0 (I_2 - I_1),$$

$$\oint_{C_3} \vec{B} \cdot d\vec{l} = \mu_0 (-I_2 - I_1).$$